Effect of Loss and Frequency Dispersion on the Performance of Microstrip Directional Couplers and Coupled Line Filters

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Abstract—**The effect of ohmic and dielectric losses, conductor thickness, and frequency dispersion on the performance of edge-coupled microstrip directional couplers and interdigital filters have been determined in this short paper. The odd- and even-mode attenuation constants due to ohmic losses in the conductor have been calculated using Wheeler's inductance formula. The theoretical results for the characteristic impedance and propagation constants are in good agreement with the experimental results of Napoli and Hughes. Among the parameters that can be calculated from this theory are the isolation, directivity, and coupling coefficients of lossy directional couplers and the midband insertion loss of interdigital filters.**

I. INTRODUCTION

Coupled line parameters for designing microstrip components such as directional couplers [1] and bandpass filters [2], [3] are generally obtained from the quasi-static theory for microstrip lines developed by several authors [4]–[6]. To reduce the difficulties associated with a rigorous analysis, these theories make a number of simplifying assumptions. 1) The coupled microstrip lines are assumed to be of "zero" thickness. 2) The ohmic and dielectric losses in the lines are assumed to be negligible. 3) A quasi-TEM mode of propagation is assumed, and frequency dispersive effects are ignored. In reality, however, the finite thickness of the conducting lines can have an appreciable effect on the coupling capacitances and on the propagation constants of the odd and even modes. Losses in microstrip lines are not negligible and set limiting bounds on the isolation and VSWR of directional couplers and on the midband insertion loss of coupled line filters. At higher microwave frequencies, the TEM mode approximation is no longer valid, and the onset of the higher order modes makes the lines dispersive.

A knowledge of all these effects is necessary to make an accurate design of coupled microstrip components to meet prescribed specifications. In this short paper, an attempt has been made to estimate approximately the effect of loss, conductor thickness, and frequency dispersion on the performance of microstrip edge-coupled directional couplers and interdigital filters.

II. LOSSES IN COUPLED MICROSTRIP LINES

Although several authors have investigated, both theoretically and experimentally, the losses in a single microstrip line [7] and also in coupled (balanced) striplines [8], the problem of losses in coupled microstrip lines appears to have received only scant attention so far [9]. In this short paper, the odd- and even-mode attenuation constants due to ohmic losses in the coupled microstrip lines have been calculated using Wheeler's incremental inductance formula [7], [8] (see Fig. 1). The odd-mode attenuation constant α_{OC} is given by

$$\alpha_{OC} = \frac{R_S}{2Z_{\text{odd}}\eta} \cdot \frac{\delta(\epsilon_O Z_{\text{odd}})}{\delta n} \quad \text{(nepers/unit length)} \tag{1a}$$

where

$$\frac{\partial(\epsilon_O Z_{\text{odd}})}{\delta n} = \frac{2}{H}\left\{\left[1 - \frac{S}{2H}\right]\frac{\partial(\epsilon_O Z_{\text{odd}})}{\partial(S/H)} - \left[1 + \frac{T}{2H}\right]\frac{\partial(\epsilon_O Z_{\text{odd}})}{\partial(T/H)} - \left[1 + \frac{A}{2H}\right]\frac{\partial(\epsilon_O Z_{\text{odd}})}{\partial(A/H)}\right\}. \tag{1b}$$

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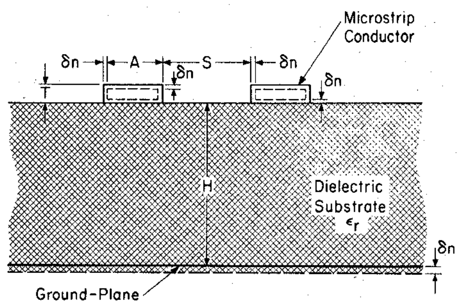


Fig. 1. Ohmic-loss calculations in coupled microstrip lines.

TABLE I
ODD- AND EVEN-ATTENUATION COEFFICIENTS α_{CO} AND α_{CE} DUE TO SKIN EFFECT LOSSES IN COUPLED MICROSTRIP TRANSMISSION LINES

Case 1 A/H = 0.785; S/H = 2B/H = 0.304; T/H = 0.0047; ϵ_S = 10.4

Frequency	8 GHz	4 GHz	0.9GHz
odd mode α_{OC}(db/cm)	0.072	0.051	0.024
even mode α_{EC}(db/cm)	0.032	0.023	0.010

Case 2 A/H = 0.870; S/H = 0.212; T/H = 0.0047; ϵ_S = 9.5

Frequency	8 GHz	4 GHz	0.9 GHz
odd mode α_{OC}(db/cm)	0.088	0.062	0.029
even mode α_{EC}(db/cm)	0.028	0.020	0.009

Case 3 A/H = 0.870; S/H = 2B/H = 0.162; T/H = 0.0047; ϵ_S = 9.5

Frequency	8 GHz	4 GHz	0.9 GHz
odd mode α_{OC}(db/cm)	0.101	0.072	0.034
even mode α_{EC}(db/cm)	0.028	0.020	0.009

Similarly, the even-mode attenuation constant is given by

$$\alpha_{EC} = \frac{R_S}{2Z_{\text{even}}\eta} \cdot \frac{\partial(\epsilon_E Z_{\text{even}})}{\partial n} \quad \text{(nepers/unit length)} \tag{2a}$$

where

$$\frac{\partial(\epsilon_E Z_{\text{even}})}{\partial n} = \frac{2}{H}\left\{\left[1 - \frac{S}{2H}\right]\frac{\partial(\epsilon_E Z_{\text{even}})}{\partial(S/H)} - \left[1 + \frac{T}{2H}\right]\frac{\partial(\epsilon_E Z_{\text{even}})}{\partial(T/H)} - \left[1 + \frac{A}{2H}\right]\frac{\partial(\epsilon_E Z_{\text{even}})}{\partial(A/H)}\right\}. \tag{2b}$$

In the preceding equations, R_S equals surface resistivity of copper $= 8.26 \times 10^{-3} f^{1/2}\ \Omega$, where f equals frequency in gigahertz. A and T are the width and thickness of the lines, and S is the spacing between the lines. H is the thickness of the dielectric substrate. ϵ_O, ϵ_E and Z_{odd} and Z_{even} are the odd- and even-mode effective dielectric constants and characteristic impedances, respectively. ϵ_S equals the relative dielectric constant of the substrate. η equals the free space impedance (377 Ω).

The partial derivatives of the characteristic impedances Z_{odd} and Z_{even} with respect to S/H, T/H, and A/H were calculated using the theory and computer program developed by Okugawa and Hagiwara [10]. Table I shows the calculated odd- and even-mode attenuation constants due to ohmic losses for three different sets of coupled lines. The losses, calculated at 8, 4, and 0.9 GHz, are shown in this table.

The odd-mode attenuation constant α_{OC} is always higher than the even-mode attenuation constant α_{EC}. α_{OC} is also more sensitive to changes in spacing "S" between the lines than is α_{EC}.

The dielectric losses α_{OD} and α_{ED} in the coupled lines for the odd and even modes are given by the expression

$$\alpha_{OD} = 91.0 \frac{\epsilon_S}{(\epsilon_O)^{1/2}} \left(\frac{\epsilon_O - 1}{\epsilon_S - 1} \right) f \tan \delta \tag{3a}$$

and

$$\alpha_{ED} = 91.0 \frac{\epsilon_S}{(\epsilon_E)^{1/2}} \left(\frac{\epsilon_E - 1}{\epsilon_S - 1} \right) f \tan \delta \tag{3b}$$

where $\tan \delta$ is the loss tangent of the microstrip dielectric substrate. The total attenuation constants α_O and α_E for the odd and even modes are given by $\alpha_O = \alpha_{OC} + \alpha_{OD}$ and $\alpha_E = \alpha_{EC} + \alpha_{ED}$.

III. EFFECT OF CONDUCTOR THICKNESS AND FREQUENCY DISPERSION ON THE PARAMETERS OF COUPLED MICROSTRIP LINES

A rigorous theory for evaluating dispersive effects in *thick* coupled microstrip lines is prohibitively difficult. However, reasonable results can be obtained by first calculating the quasi-static odd- and even-mode impedances and propagation constants using the theory of Okugawa [10] and then applying appropriate corrections for frequency dispersion as suggested by Getsinger [11]. Table II shows the comparison of the theoretical results for Z_{odd}, Z_{even}, λ_{gO}, and λ_{gE} calculated by using this procedure with the experimental results measured by Napoli and Hughes [1]. For loosely coupled lines, the theoretical results and the measured values appear to agree to within 5 percent. However, fairly large discrepancies are noticed for the even-mode impedance Z_{even} of very tightly coupled lines ($S/H \leq 0.12$).

Fig. 2(a) and (b) shows the effect of conductor thickness and frequency dispersion on the parameters $Z_{\text{odd}}/Z_{\text{even}}$, $\lambda_{gO}/\lambda_{gE}$, and the coupling coefficient C (dB) $= 20 \log_{10} [(Z_{\text{even}} - Z_{\text{odd}})/(Z_{\text{even}} + Z_{\text{odd}})]$. The calculations indicate that the conductor thickness appears to have a greater effect on the parameters of the odd mode, whereas the frequency dispersion affects the even mode more strongly.

IV. EFFECT OF LOSS AND FREQUENCY DISPERSION ON EDGE-COUPLED MICROSTRIP DIRECTIONAL COUPLERS

A scattering matrix analysis was made to estimate the effects of losses and frequency dispersion on the performance of edge-coupled microstrip directional couplers. The theory, with appropriate modifications, is similar to that proposed by Brenner [12]. The scattering matrix coefficients for the lossy quarter-wave microstrip directional coupler are given by

$$S_{11} = 1 - (Z_O/2)(I_1 + I_2 + I_3 + I_4) \tag{4a}$$

$$S_{12} = -(Z_O/2)(I_1 + I_2 - I_3 - I_4) \tag{4b}$$

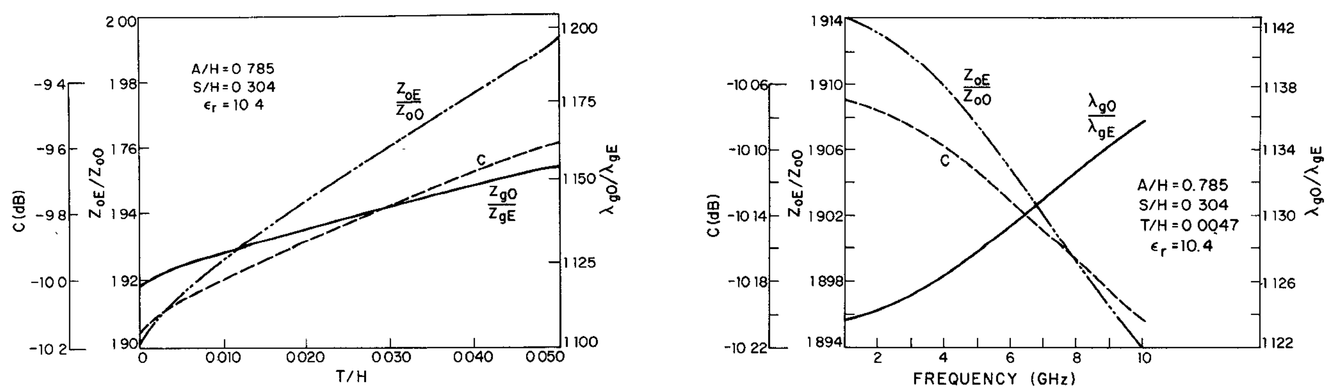
$$S_{13} = -(Z_O/2)(I_1 - I_2 - I_3 + I_4) \tag{4c}$$

$$S_{14} = -(Z_O/2)(I_1 - I_2 + I_3 - I_4) \tag{4d}$$

TABLE II

COMPARISON BETWEEN THEORY AND EXPERIMENTAL RESULTS

Coupled Transmission Line Parameters	Experimental Results of Napoli & Hughes ϵ_S=10.4 Frequency = 8 GHz	Theoretical Results (effect of conductor thickness & frequency dispersion included) ϵ_S=10.4; 8 GHz	Percentage Error (Theory) - (Expt) / (Expt) x 100
Case 1 A/H = 0.863; S/H = 2B/H = 0.528; T/H = 0.0047			
Z_{even}	58.0	60.8	+4.8%
Z_{odd}	38.0	38.6	+1.5%
λ_{gE}/λ_o	0.356	0.357	+0.3%
λ_{go}/λ_o	0.414	0.406	-1.9%
$\lambda_{go}/\lambda_{gE}$	1.163	1.133	-2.5%
Case 2 A/H = 0.960; S/H = 2B/H = 1.000; T/H = 0.0047			
Z_{even}	52.1	53.62	+3.1%
Z_{odd}	40.0	41.65	+4.1%
λ_{gE}/λ_o	0.355	0.354	-0.1%
λ_{go}/λ_o	0.403	0.400	-0.7%
$\lambda_{go}/\lambda_{gE}$	1.135	1.126	+0.8%
Case 3 A/H = 0.785; S/H = 2B/H = 2B/H = 0.304; T/H = 0.0047			
Z_{even}	64.0	67.2	+5.0%
Z_{odd}	34.0	35.4	+4.0%
λ_{gE}/λ_o	0.357	0.360	+0.8%
λ_{gE}/λ_o	0.418	0.409	-2.1%
$\lambda_{go}/\lambda_{gE}$	1.17	1.13	-3.4%
Case 4 A/H = 0.368; S/H = 2B/H = 0.120; T/H = 0.0047			
Z_{even}	86.0	102.0	+18.6%
Z_{odd}	35.0	35.6	+1.7%
λ_{gE}/λ_o	0.372	0.376	+1.0%
λ_{go}/λ_o	0.424	0.416	-1.8%
$\lambda_{go}/\lambda_{gE}$	1.14	1.10	-3.5%
Case 5 A/H = 0.176; S/H = 2B/H = 0.084; T/H = 0.0047			
Z_{even}	118.0	135.0	+14.4%
Z_{odd}	37.5	39.4	+5.0%
λ_{gE}/λ_o	0.383	0.386	+0.8%
λ_{go}/λ_o	0.433	0.420	-3.0%
$\lambda_{go}/\lambda_{gE}$	1.13	1.09	-3.5%



(a) (b)

Fig. 2. (a) Variation of Z_{0E}/Z_{0O}, $\lambda_{gO}/\lambda_{gE}$, and C with conductor thickness (T/H). (b) Variation of Z_{0E}/Z_{0O}, $\lambda_{gO}/\lambda_{gE}$, and C with frequency.

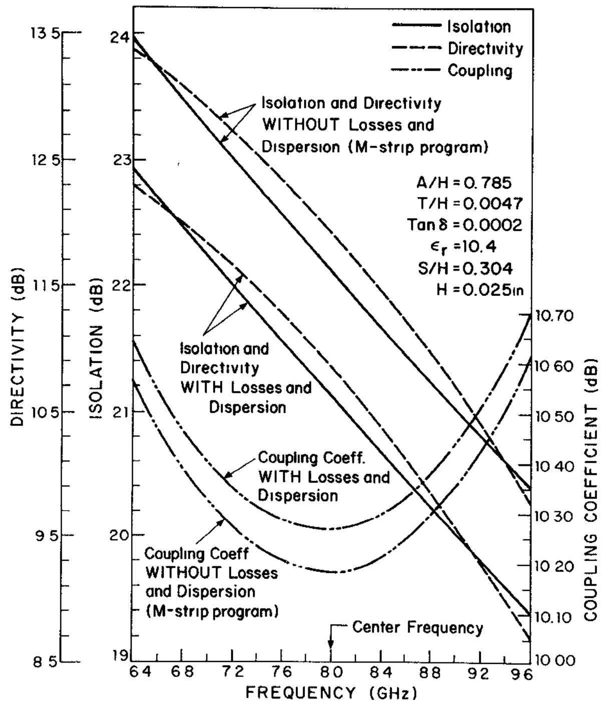


Fig. 3. Effect of loss and frequency dispersion on the isolation, directivity, and coupling coefficient of a microstrip directional coupler.

where

$$I_1 = \frac{E}{Z_{LE}\coth(\gamma_E(l/2)) + Z_O} \tag{5a}$$

$$I_2 = \frac{E}{Z_{LE}\tanh(\gamma_E(l/2)) + Z_O} \tag{5b}$$

$$I_3 = \frac{E}{Z_{LO}\tanh(\gamma_O(l/2)) + Z_O} \tag{5c}$$

$$I_4 = \frac{E}{Z_{LO}\tanh(\gamma_O(l/2)) + Z_O} \tag{5d}$$

where $\gamma_E = \alpha_E + j\beta_E$ and $\gamma_O = \alpha_O + j\beta_O$ are the even- and odd-mode propagation constants. $\beta_E = 2\pi/\lambda_{gE}$ and $\beta_O = 2\pi/\lambda_{gO}$ are the even- and odd-mode phase constants. Z_{LO} and Z_{LE} are the complex odd- and even-mode impedances for the lossy coupled lines:

$$Z_{LO} \simeq Z_{\text{odd}}\left\{1 - j\frac{\alpha_{OC}}{\beta_O} + j\frac{\alpha_{OD}}{\beta_O}\right\} \tag{6a}$$

and

$$Z_{LE} \approx Z_{\text{even}}\left\{1 - j\frac{\alpha_{EC}}{\beta_E} + j\frac{\alpha_{ED}}{\beta_E}\right\}. \tag{6b}$$

Z_{odd} and Z_{even} are the odd- and even-mode impedances for the lossless lines; the derivation of (6a) and (6b) is described in [15]. In (4) and (5), Z_O is the source impedance and $4E$ is the source voltage. $l = \frac{1}{4}[(\lambda_{gO} + \lambda_{gE})/2]$ is the physical length of the coupling section, at the design center frequency.

Fig 3 shows the calculated coupling coefficient, directivity, and isolation as a function of frequency for a 10-dB microstrip directional coupler. The center design frequency of this coupler was 8 GHz. The corresponding values calculated from the M-strip program of Bryant and Weiss [4] are also plotted in the same figure for comparison. The M-strip program does not take into account the conductor thickness, losses, or the frequency dispersive effects. From Fig. 3 it is noticed that the maximum differences in the results calculated from these two theories occur at the midband (design) frequency. The directivity of the coupler is primarily determined by the difference between λ_{gE} and λ_{gO}. The losses and the difference between λ_{gE} and λ_{gO} also have an appreciable effect on the midband isolation and coupling coefficient. The results obtained from this theory agree qualitatively with the experimental measurements made by other authors [5], [13].

V. EFFECT OF LOSS AND FREQUENCY DISPERSION ON THE PERFORMANCE OF A MICROSTRIP INTERDIGITAL FILTER

An analysis has also been made to determine the effect of loss and frequency dispersion on a microstrip interdigital filter. The analysis is similar to that applied by Jones and Bolljahn [14] for studying stripline filters. However, in the microstrip problem, the odd- and even-mode propagation constants as well as attenuation constants are unequal. The image impedance Z_I and the image transfer constant $\cosh\phi$ for a single-section microstrip interdigital filter are given by

$$Z_1 = \frac{2Z_{LO}\cdot Z_{LE}}{\{Z_{LE}^2 + Z_{LO}^2 + 2Z_{LO}\cdot Z_{LE}(\coth\gamma_E l\cdot\coth\gamma_O l + \operatorname{csch}\gamma_E l\cdot\operatorname{csch}\gamma_O l)\}^{1/2}} \tag{7a}$$

and

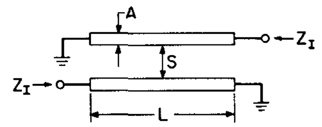
$$\cosh\phi = \frac{Z_{LE}\coth\gamma_O l + Z_{LO}\coth\gamma_E l}{Z_{LE}\operatorname{csch}\gamma_O l - Z_{LO}\operatorname{csch}\gamma_E l} \tag{7b}$$

where

$$\gamma_E = \alpha_E + j\beta_E \quad\text{and}\quad \gamma_O = \alpha_O + j\beta_O. \tag{7c}$$

l is the length of the coupled lines calculated for the design center frequency and is given by [2]

$$l = \frac{\pi/2}{(\beta_E + \beta_O)/2 + [(Z_{\text{even}} - Z_{\text{odd}})/Z_{\text{even}} + Z_{\text{odd}})][(\beta_E - \beta_O)/2]}. \tag{7d}$$



MICROSTRIP INTERDIGITAL FILTER

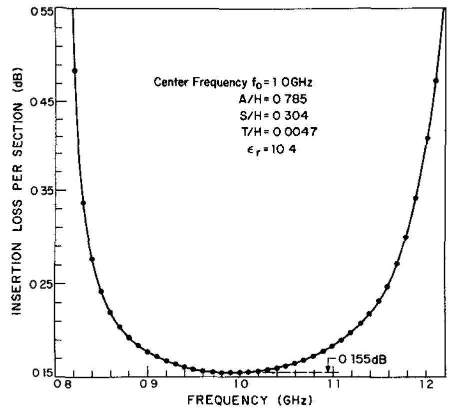


Fig. 4. Insertion loss per section of microstrip interdigital filter.

The single-section insertion loss versus frequency of a microstrip interdigital filter is shown in Fig. 4. The center frequency for this filter is 1 GHz. Calculations using the formula, as given previously, indicate that this filter has a midband insertion loss of 0.155 dB. Losses in any given filter structure may then be calculated in terms of loss due to individual sections as a function of frequency. Several types of microstrip filter structures can be analyzed using this method.

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